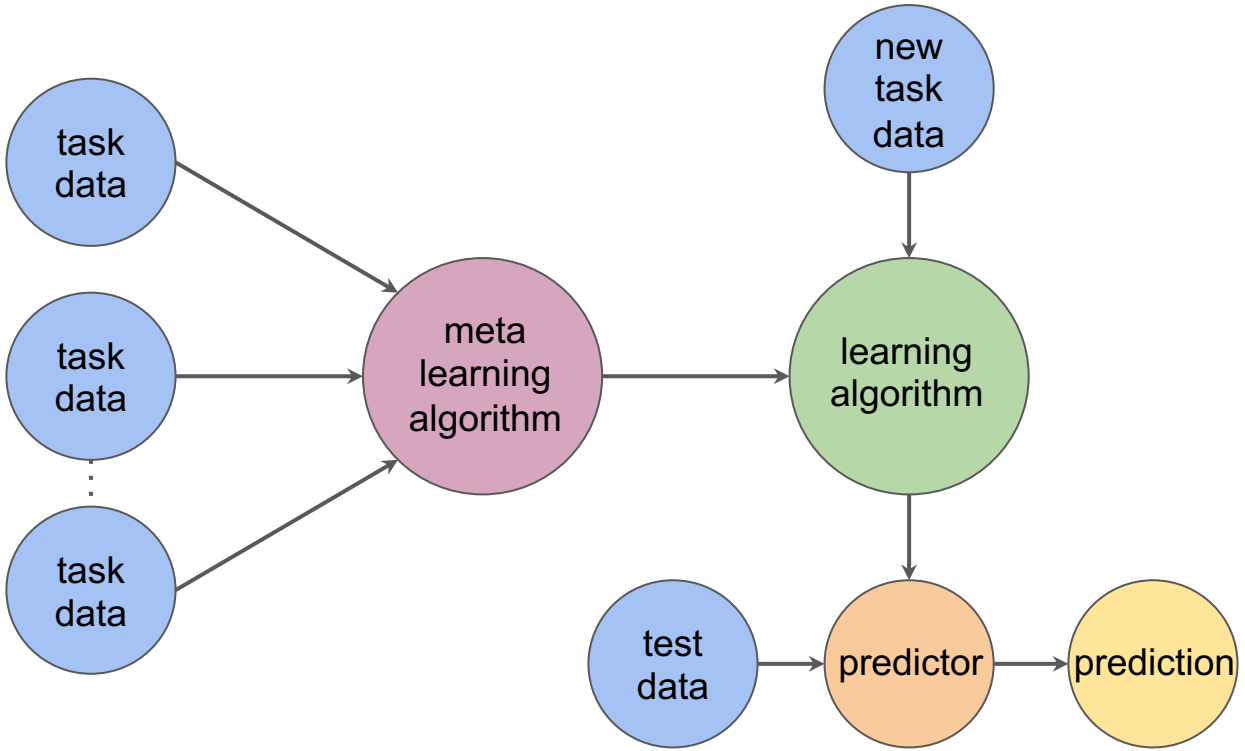
Meta Learning

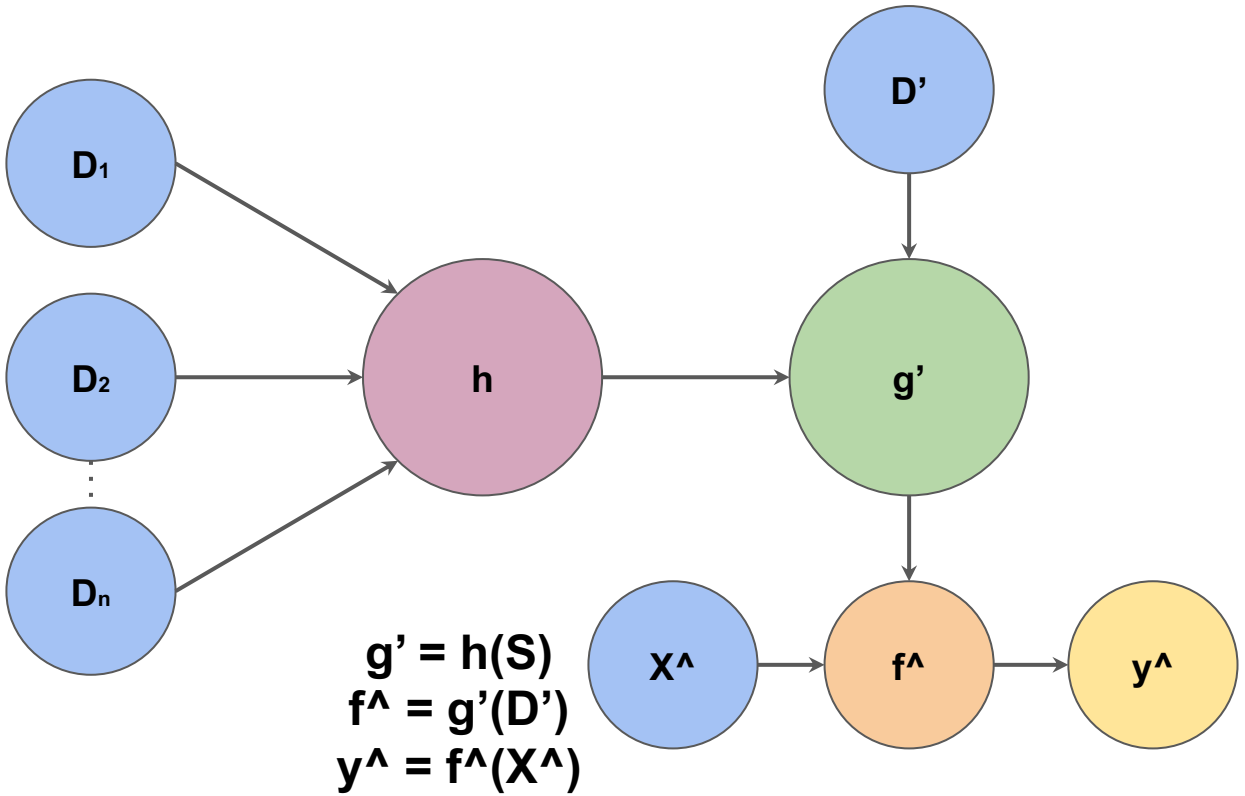
MIT
Iddo Drori, Fall 2020

Meta Learning

task = data splits, priors

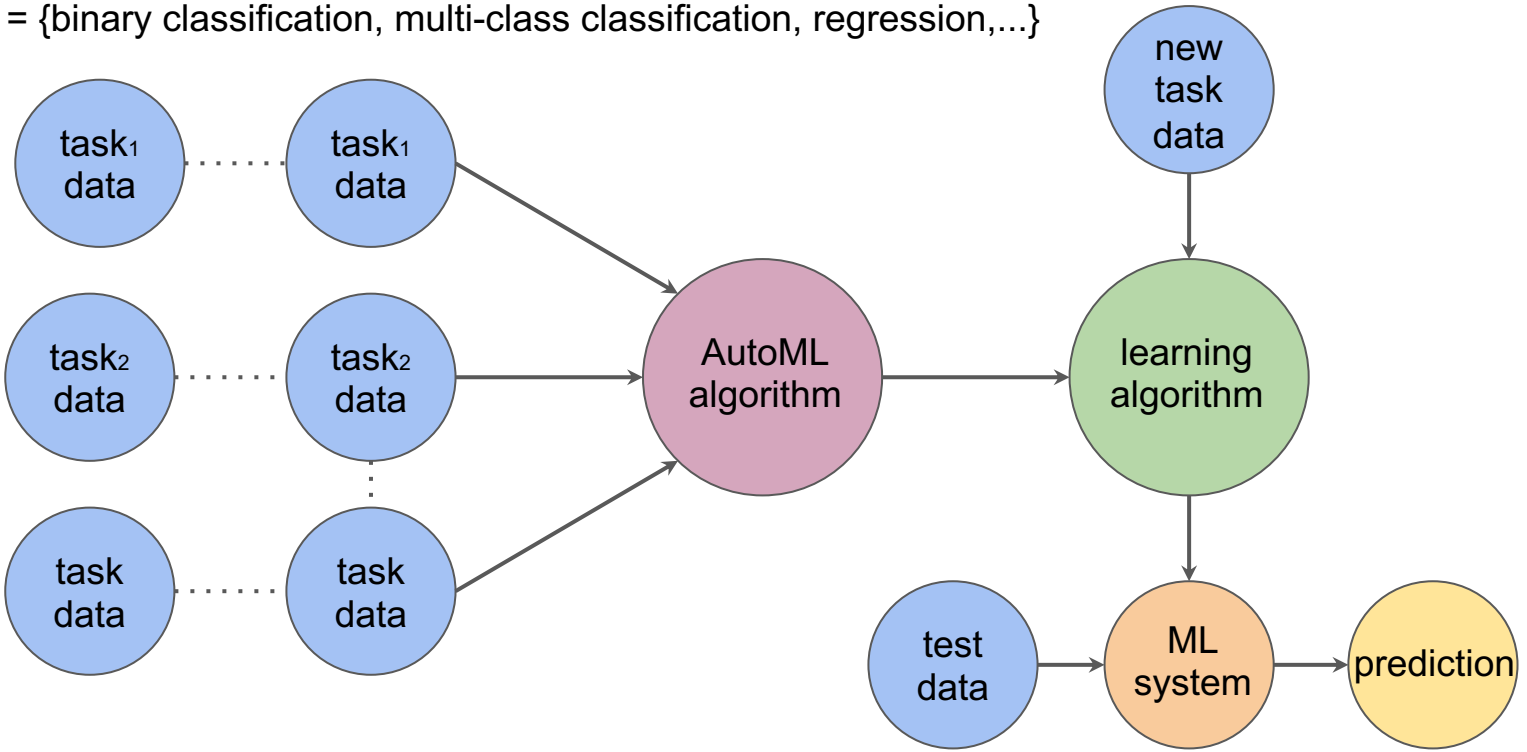


Meta Learning

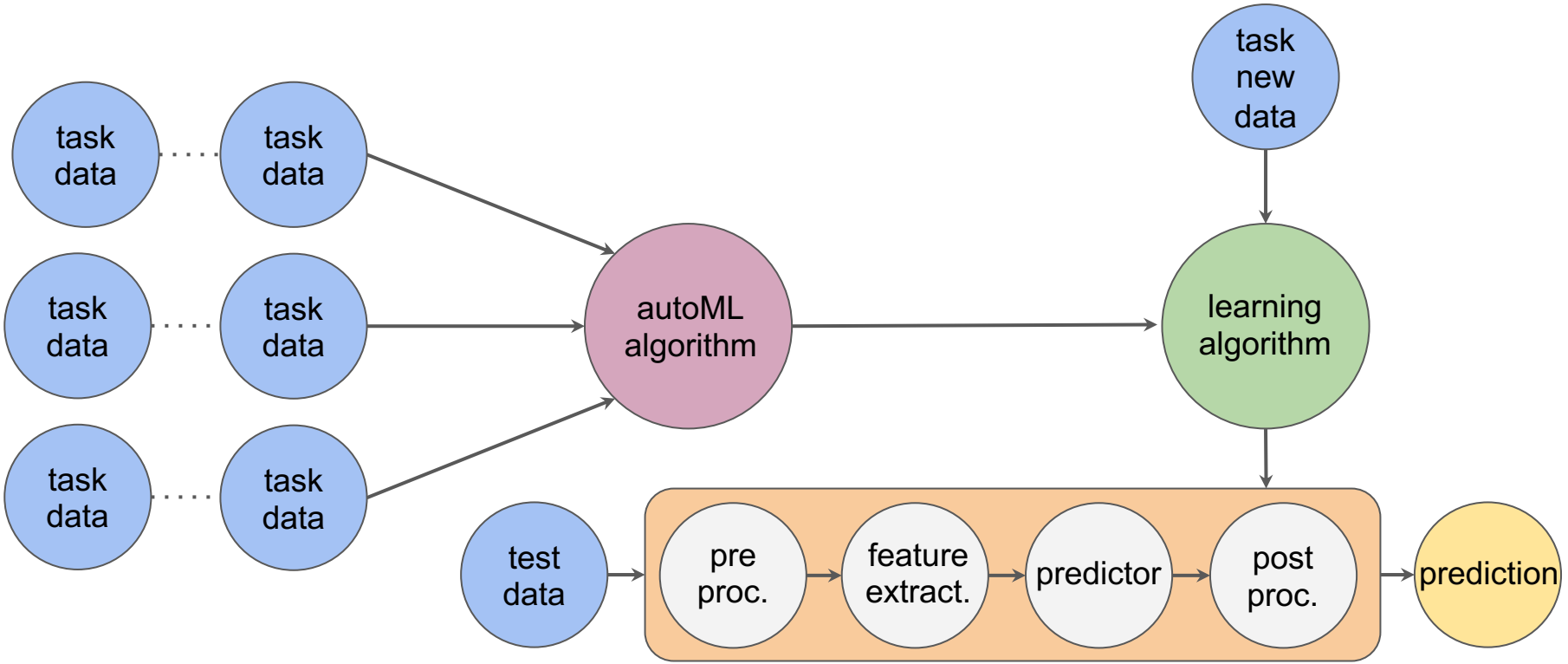


Automated Machine Learning

tasks = {binary classification, multi-class classification, regression,...}



Automated Machine Learning (AutoML)



Machine Learning Systems

ML System

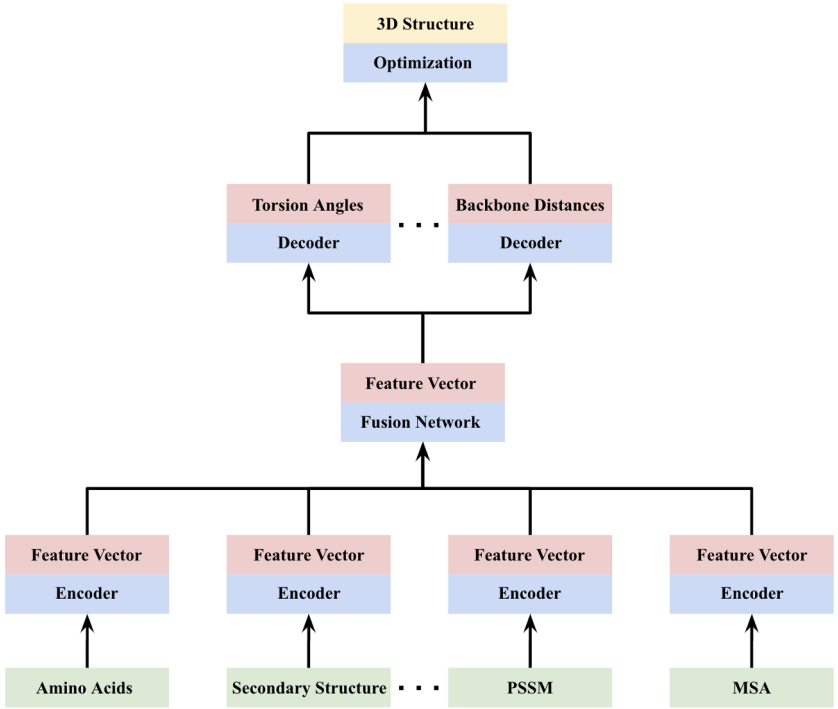


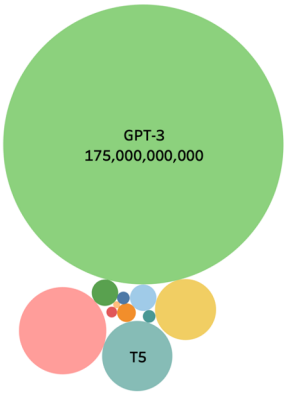
Figure source: Accurate protein structure prediction by embeddings and deep learning representations, Drori et al, 2019.

Transformers

Transformers

- Text
- Task agnostic architecture
- Task specific datasets of 1-100k samples for fine tuning

  or
- Few shot learning



Fine Tuning

- Examples for specific task
- Gradient updates

Meta Learning in Language Models

- Humans don’t require so many examples for new tasks instead use a handful
- Avoid collecting domain specific datasets and fine tuning for new tasks
- Broad learning during training
- Adapt to tasks at runtime

Few-Shot Learning

- Only examples at runtime: number of examples k, 1, 0
- K examples of task
  - Translate English to Chinese
  - how are you -> ni hao ma
  - 1,2,3 -> yi, er, san
- 1 example of task
  - Translate English to Chinese
  - how are you -> ni hao ma
- No examples, language description
  - Translate English to Chinese

Math Word Problems and Question Answering

Math Word Problems and Question Answering

- Question

  “At the fair Adam bought 13 tickets. After riding the ferris wheel he had 4 tickets left. If each ticket cost 9 dollars, how much money did Adam spend riding the ferris wheel?

- Answer

  81

Math Word Problems Approaches

- Template-based methods
- Prediction of operators and operands
- Search space of binary expression trees
- Deep neural networks
- Reinforcement learning, building expression trees

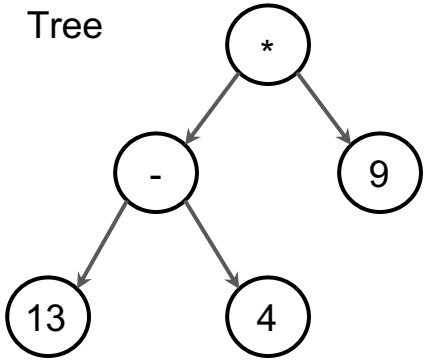
Expression Tree

- Question

  “At the fair Adam bought 13 tickets. After riding the ferris wheel he had 4 tickets left. If each ticket cost 9 dollars, how much money did Adam spend riding the ferris wheel?

- Answer

  81



Expression

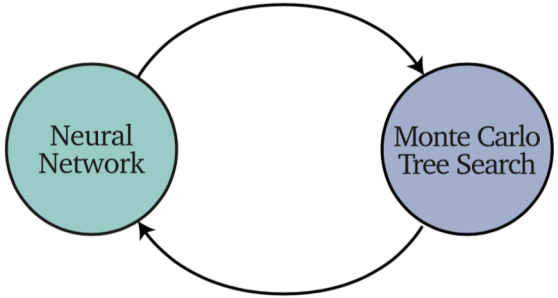
(13 - 4) * 9 = 81

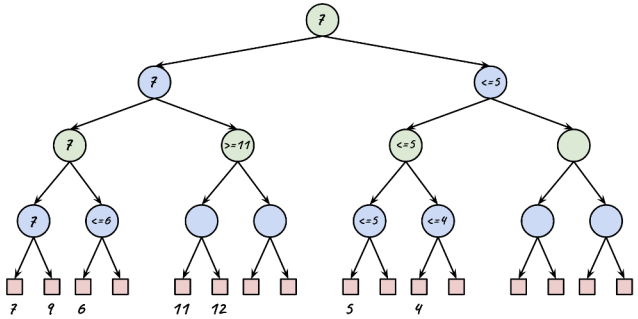
x=13 y=4 z=9
(x op1 y) op2 z
x y op1 z op2

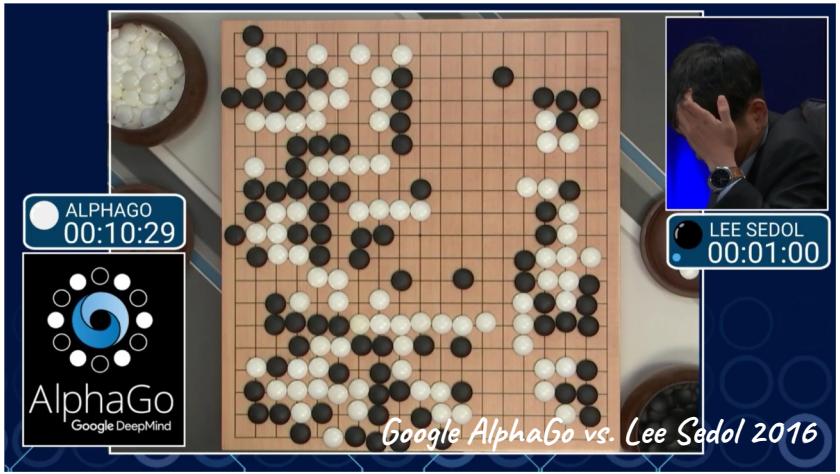
Prediction

- Probability over operators and operands
- Probability over trees

20 Years of Superhuman Game Playing

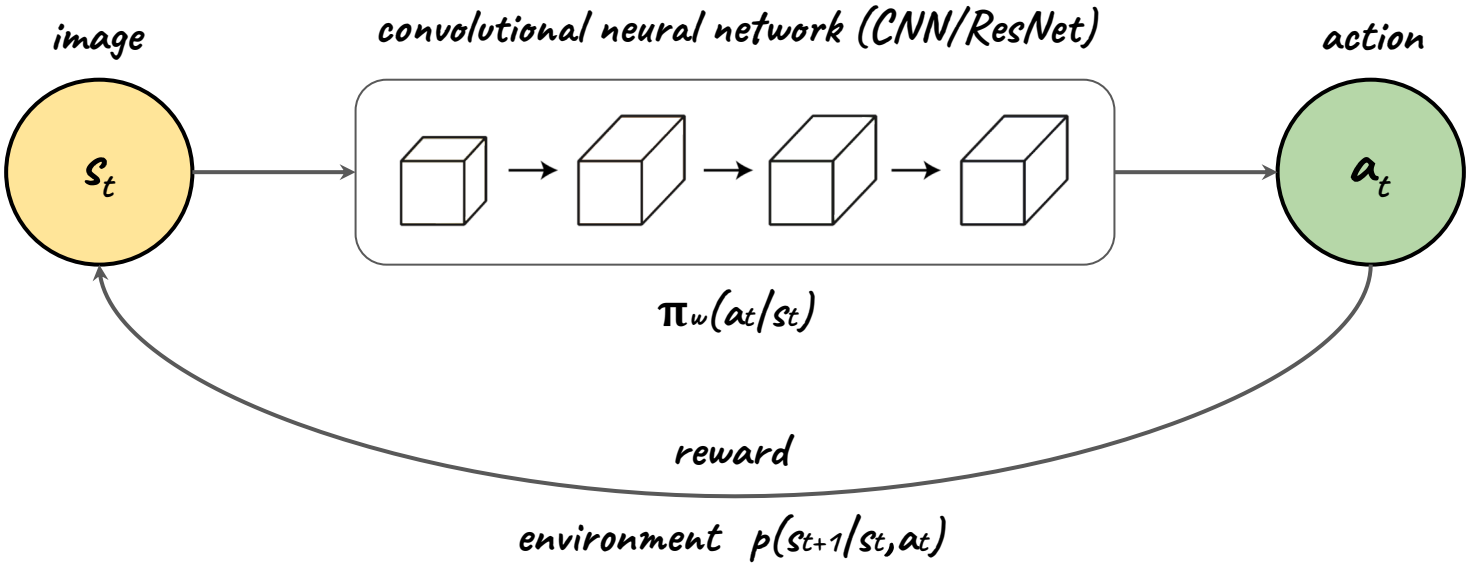






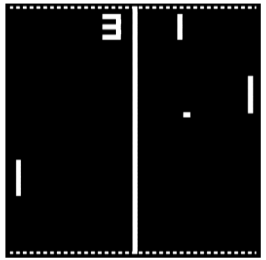
Deep Reinforcement Learning

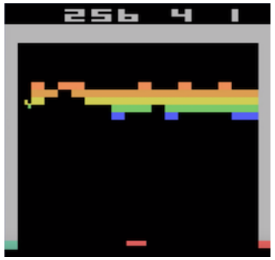
- Deep neural network represents policy, value function, model
- Optimize loss function by stochastic gradient descent

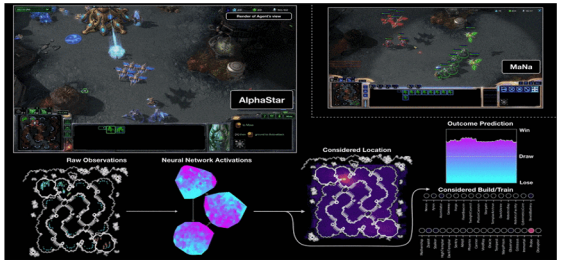


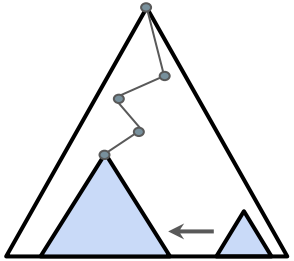
Deep Reinforcement Learning Applications

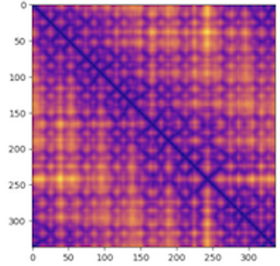
- Video games
- Board games
- Rubik's cube
- Protein folding
- Dialogue synthesis
- Automatic machine learning
- Robot control
- Self driving cars

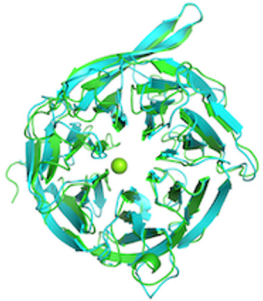




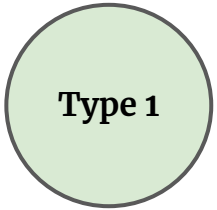








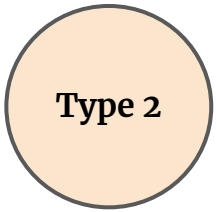
Motivation: Dual Process Theory



Fast

Autonomous

May not require working memory



Slow

Involves mental simulation and decoupling

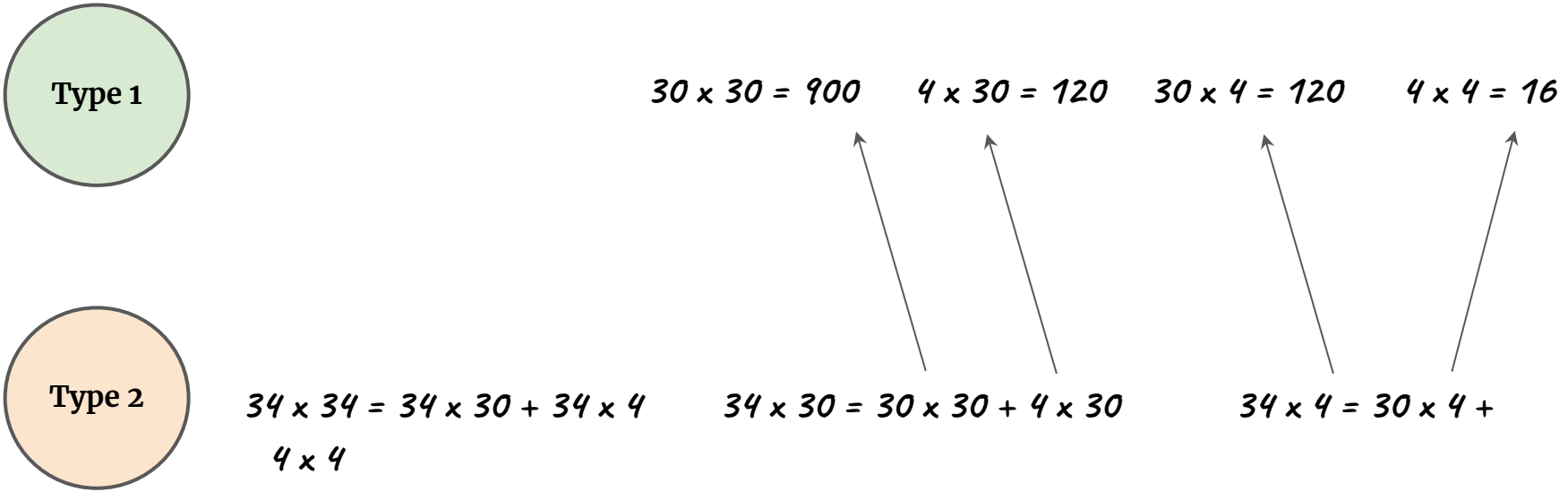
Requires working memory

Daniel Kahneman, Thinking Fast and Slow, 2011

Dual Process Theory: Simple Analogy

$34^2 = ?$

Dual Process Theory: Simple Analogy



Dual Process Theory: Simple Analogy

1156

Dual Process Theory: Simple Analogy

Q: Second time, what is 34^2?

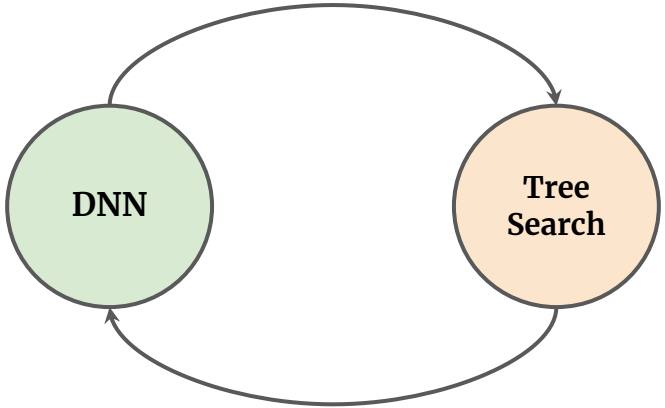
A: *1156* right away, since its now type 1, so we'll keep the network which knows this rather than use previous network.

Q: Next, what is 34^4, use 34^2 etc.

Dual process iteration with self play.

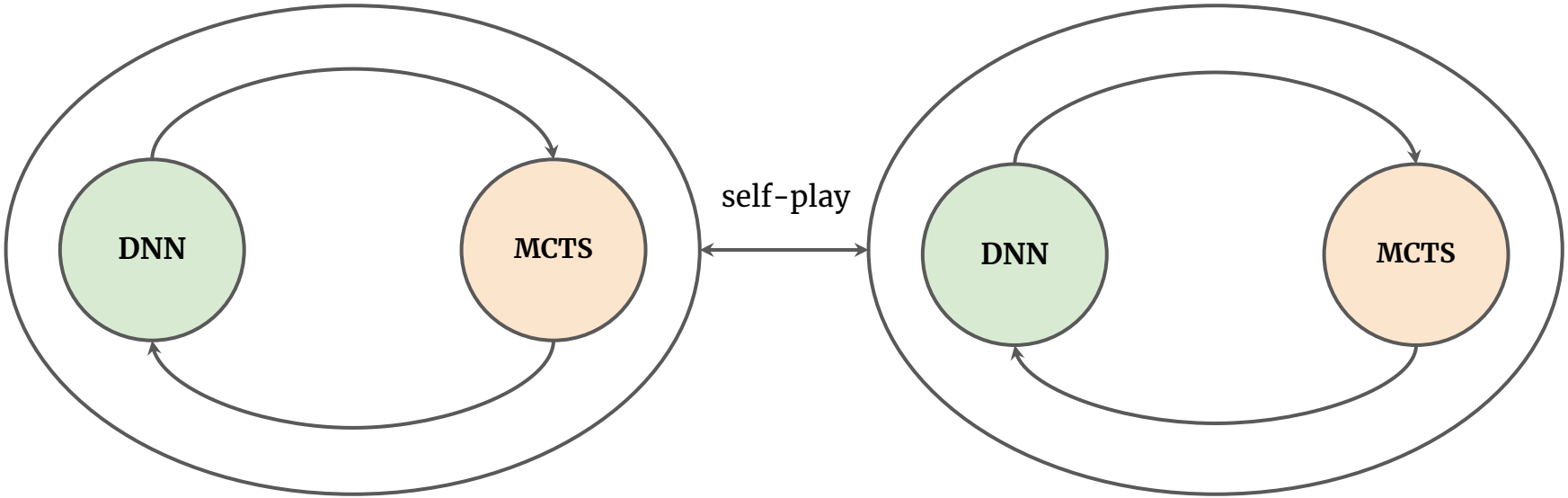
Expert Iteration

$$f_\theta(s) = \big(P(a|s), v(s)\big)$$



Anthony et al., Thinking fast and slow with deep learning and tree search, NeurIPS 2017.

AlphaZero



Mastering chess and Shogi by self-play with a general reinforcement learning algorithm, Silver et al, 2018

Neural Network Loss Function

- Minimize loss function

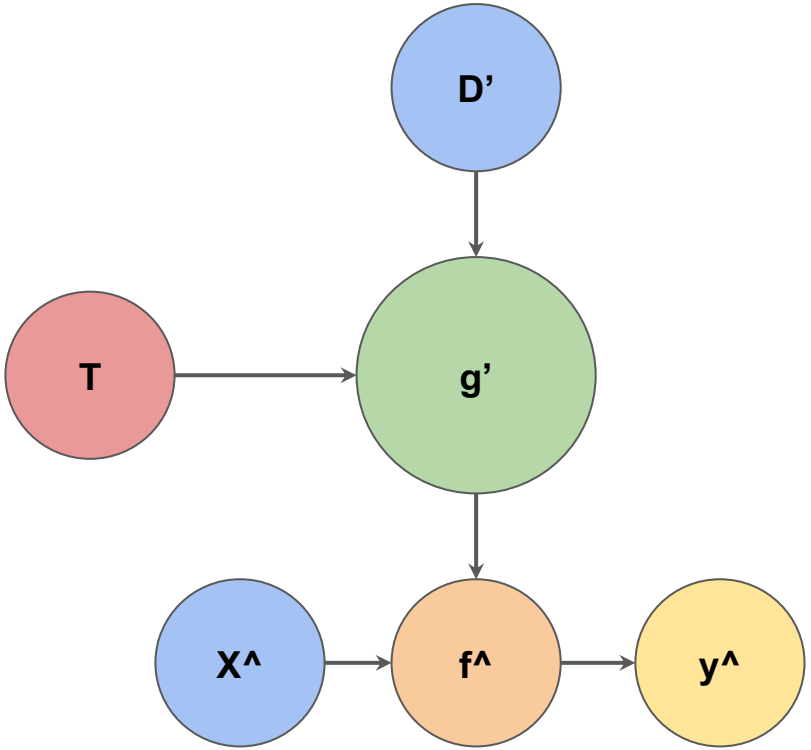
$$L(\theta) = -\pi \log p + (v - e)^2 + \alpha \|\theta\|^2$$

θ	Neural network parameters
v	Neural network predicted value
e	Actual value
p	Neural network predicted probabilities
π	Actual search probabilities

Math Question Answering using a Transformer and Reinforcement Learning

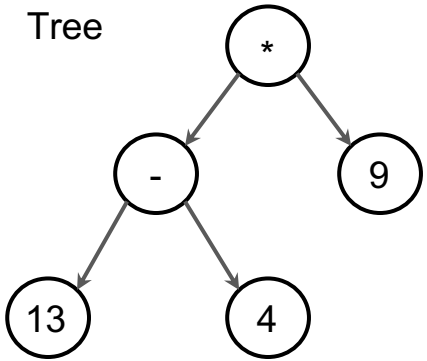
- **T**ransformer
- **g’** is RL

f^ = g’(D’,T)
y^ = f^(X^)



Reinforcement Learning

- State: graph, tree, expression
- Actions: selected operator and operands
- Reward: correct action or expression evaluation



Expression

(13 - 4) * 9 = 81

Meta 6.036 Lab 1

- Machine Learning prerequisite
- Max cap of 50 students

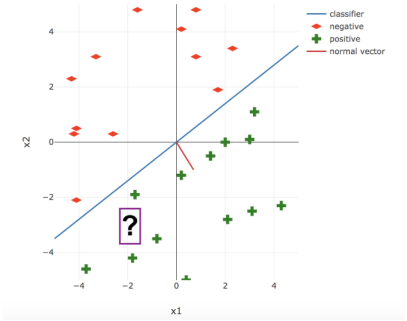
Meta 6.036 Lab 1

Meta 6.036 Lab 1

1.1) Visual Intuition

We have some data that we know falls into two categories: positive (shown as a green plus sign) and negative (shown as a red minus sign). We want to predict whether new data points are positive or negative. The image below shows a linear classifier (the blue line) that our machine found, which perfectly classifies the existing data (though in practice, it's rare to perfectly classify real world data).

Note: Here we use x_1 and x_2 as axes instead of the typical x, y axis



A) A new data point is shown in the plot above as a question mark. What category will our linear classifier predict this new data point to be in? Positive or Negative?

The new point will be classified as:

Positive

Negative

Save | Submit | View Answer | Ask for Help 100.00%

As staff, you are always allowed to submit. If you were a student, you would see the following:
You have infinitely many submissions remaining.

B) Is it guaranteed that this prediction is correct?

Massachusetts Institute of Technology

Meta 6.036 Lab 1

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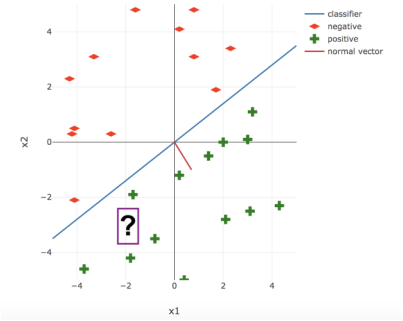

A) A new data point is shown in the plot above as a question mark. What category will our linear classifier predict this new data point to be in? Positive or Negative?

The new point will be classified as:

- (•) Positive
- ( ) Negative

Save | Submit | View Answer | Ask for Help 100.00%

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Meta 6.036 Lab 1

- Transformer
- CNN

Meta 6.036 Lab 1

1) Linear Classifiers

Machine Learning is about making decisions or predictions based on data. Often times, we will use models to help us make good predictions or decisions. Models are, essentially, fancy functions. Given new input data, models output something that helps us (or a computer) make predictions/make decisions.

The beauty of machine learning is that, rather than building a model ourselves that allows us to make predictions/decisions, we can make a machine *train* a model that makes good predictions and decisions for our use case!

Binary Linear Classifier

The first model we look at in this course is a **binary linear classifier.** As you can read about in the lecture notes, a *binary linear classifier* is a simple, yet powerful type of model (which is, again, a fancy function) that is *linear* and that *classifies* data into two (*binary*) categories. In other words, given some new data, it will predict which category that data falls into.

How does this work? Let's find out!

Meta 6.036 Lab 1

- Transformer

Meta 6.036 Lab 1

D) Graph the line $x_1 + 2 = 2x_2$ on a x_1, x_2 plane. Then, plot the vector $\theta = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$. What do you notice about the vector in relation to the slope of the linear function it represents?

Meta 6.036 Lab 1

- Transformer
- Expression tree
- Meta learning

Meta 6.036 Lab 1

1.3) Predicting

So now we are familiar with how to represent a linear function as an array θ and an offset θ_0. We also know from (A) and (B) that in theory, we can use the linear function to classify points as positive and negative. But how do we decide which side is positive? How do we classify a single point? **Note: just a line is not a 2D classifier! We need to know which side of the line to designate as positive and which side is negative!**

As you may have found in (D), the vector θ is perpendicular to the line it defines. This leads to a generalization which will help us classify points as positive or negative, given θ and θ_0.

In a 2-dimensional space, our linear classifier takes the form of a 1-dimensional line that can be characterized by a scalar offset (θ_0) and a 2×1 -vector (θ) that is **normal** or **orthogonal** to the line.

Turns out...In a 3-dimensional space, our linear classifier takes the form of a 2 dimensional plane that can be characterized by a scalar offset (θ_0) and a 3×1 -vector (θ) that is orthogonal to the plane.

Notice the pattern?

In general, in a d-dimensional space, linear classifiers take the form of a $d-1$ dimensional *hyperplane* which can be characterized by a scalar offset (θ_0) and a $d \times 1$ -vector (θ) that is orthogonal to the hyperplane.

Given the offset and the θ vector, we can directly classify points as positive and negative (we don't need any more information and we don't even have to plot it as you'll see in a bit!). We will refer to classifiers and distinguish amongst classifiers using the appropriate θ vector and offset θ_0.

So how do we use θ and θ_0 to classify points?

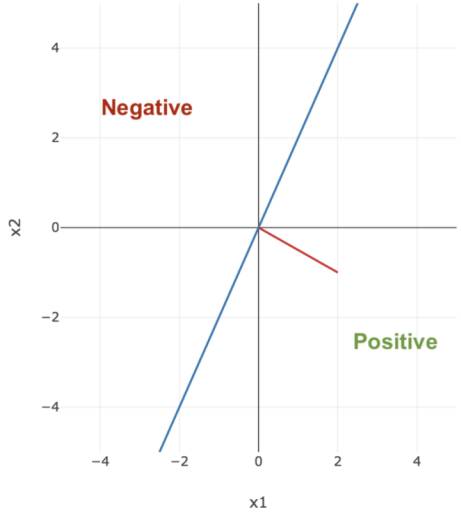
Points that fall on the side of the binary linear classifier that the normal vector points in are classified as positive. So, let's see an example!

Meta 6.036 Lab 1

- Transformer

Meta 6.036 Lab 1

Suppose $\theta = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\theta_0 = 0$. Points below this particular classifier are classified as positive and points above this classifier are considered negative.



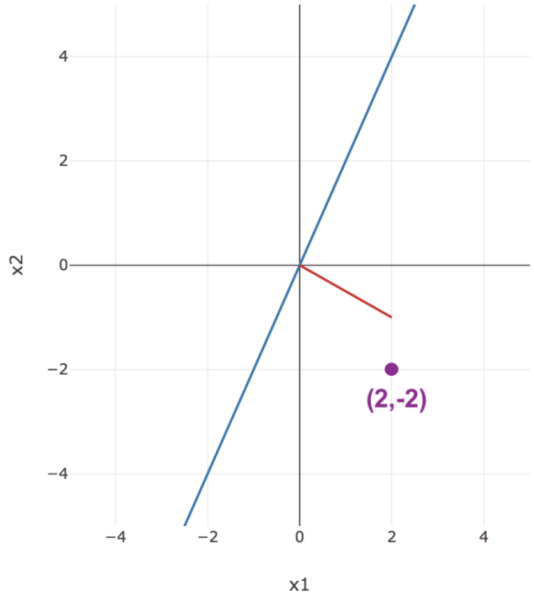
Meta 6.036 Lab 1

- Transformer, CNN
- Computational graph, expression tree
- Meta learning

Meta 6.036 Lab 1

Now suppose, we wanted to classify the point $(2, -2)$.

Looking at the graph we can see that $(2, -2)$ should be classified as positive.



Meta 6.036 Lab 1

- Transformer, CNN
- Computational graph, expression
- Meta learning

Meta 6.036 Lab 1

We can confirm this mathematically as well. We know that the equation for the line corresponding to $\theta = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\theta_0 = 0$ is $2x_1 = x_2$ (If you don't see this, derive it as shown in 1.2). Any point (d_1, d_2), for which $d_1 = 2$, will be classified as positive if $d_2 < 2d_1$. This can also be written as $0 < 2d_1 - d_2$. Thus in our case...

$$
\begin{aligned}
0 &< 2d_1 - d_2 \\
0 &< 2(2) - (-2) \\
0 &< 4 + 2 \\
0 &< 6
\end{aligned}
$$

Since the above is true, we can classify the point as positive. For the point $(-1, 1)$, is $0 < 2d_1 - d_2$? Is $0 < 2 * (-1) - 1$? It is not. Therefore, $(-1, 1)$ is classified as negative (and we can check this against the plot)! But notice how we do not require the plot to determine the classification.

Meta 6.036 Lab 1

- Transformer

Meta 6.036 Lab 1

E) Suppose we are given the classifier defined by $\theta = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$ and $\theta_0 = 0$. How will our classifier classify the point $(1, 3)$?

The classifier will classify the point (1,3) as:

Positive

Negative

Save | Submit | View Answer | Ask for Help **100.00%**

As staff, you are always allowed to submit. If you were a student, you would see the following:

You have infinitely many submissions remaining.

Meta 6.036 Lab 1

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- ( ) Negative

Save | Submit | View Answer | Ask for Help 100.00%

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You have infinitely many submissions remaining.

Meta 6.036 Lab 1

1.2) Representing as a matrix

We've established that one can use a linear binary classifier to predict a category, but we need a way to represent our classifier using an array. We need to know both how to represent the line itself, and also which side is designated as positive and which side is designated as negative.

So, let's start with the line...how do we represent a line as a matrix/array? Suppose we wish to represent the line $x_1 + 2 = 2x_2$ using matrices. We can represent this line using an array θ and an offset θ_0.

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

The equation will take the form

$$\theta^T x + \theta_0 = 0$$
$$[\theta_1, \theta_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \theta_0 = 0$$

To derive θ and θ_0 from the equation...

$$x_1 + 2 = 2x_2$$
$$x_1 - 2x_2 + 2 = 0$$
$$1x_1 + (-2)x_2 + 2 = 0$$
$$[1, -2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 2 = 0$$

Thus...

$$\theta = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \theta_0 = 2$$

Meta 6.036 Lab 1

- Transformer
- Graph

Meta 6.036 Lab 1

C) Given a line in the form $\theta^T x + \theta_0 = 0$, where θ is a *column* vector $\theta = [\theta_1, \theta_2]$, convert the equation of that line into the more familiar, $mx_1 + b = x_2$ slope-intercept form. What are m and b in terms of θ_1, θ_2, and θ_0? Enter your answers as python expressions. **Use `theta_1` for θ_1, `theta_2` for θ_2, and `theta_0` for θ_0**

Formula for m:

Check Syntax | Save | Submit | View Answer | Ask for Help 100.00%

As staff, you are always allowed to submit. If you were a student, you would see the following:

You have infinitely many submissions remaining.

Your entry was parsed as:

Formula for b:

Check Syntax | Save | Submit | View Answer | Ask for Help 100.00%

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Your entry was parsed as:

Meta 6.036 Lab 1

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Formula for m: theta_1/-theta_2

Check Syntax | Save | Submit | View Answer | Ask for Help — 100.00%

As staff, you are always allowed to submit. If you were a student, you would see the following:

You have infinitely many submissions remaining.

Your entry was parsed as:

$$\frac{\theta_1}{-\theta_2}$$

Formula for b: theta_0/-theta_2

Check Syntax | Save | Submit | View Answer | Ask for Help — 100.00%

As staff, you are always allowed to submit. If you were a student, you would see the following:

You have infinitely many submissions remaining.

Your entry was parsed as:

$$\frac{\theta_0}{-\theta_2}$$

Meta 6.036 Lab 1

- Transformer
- Graph
- Meta Learning

Meta 6.036 Lab 1

G) You are given a θ, θ_0, and data array x (where each column corresponds to one datapoint), and want to determine the classification of all points in x.

For example, if:

$$\theta = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \theta_0 = 2$$

$$x = \begin{bmatrix} 2 & 4 & 6 & 8 \\ -2 & 4 & 1 & 1 \end{bmatrix}$$

The correct output would be:

$$\begin{bmatrix} -1 & -1 & 1 & 1 \end{bmatrix}$$

Complete the following pseudocode for determining these classifications by filling the boxes with the appropriate values.

Hint: You may use a function called `sign(x)`*, which returns* -1 *for* $x < 0$, 0 *for* $x = 0$, *and* 1 *for* $x > 0$.

Take the [ ] of [ ] to get [ ].

Take the [ ] between [ ], and offset each value

in the resulting matrix with [ ] to get [ ]

Take the [ ] of [ ]

◀ | θ_0 | $x\theta^T + \theta_0$ | $x\theta + \theta_0$ | θ and x | θ^T and x | x and θ_0 ▶

Save | Submit | View Answer | Ask for Help

As staff, you are always allowed to submit. If you were a student, you would see the following:
You have infinitely many submissions remaining.

Meta 6.036 Lab 1

G) You are given a θ, θ_0, and data array x (where each column corresponds to one datapoint), and want to determine the classification of all points in x.

For example, if:

$$\theta = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \theta_0 = 2$$

$$x = \begin{bmatrix} 2 & 4 & 6 & 8 \\ -2 & 4 & 1 & 1 \end{bmatrix}$$

The correct output would be:

$$\begin{bmatrix} -1 & -1 & 1 & 1 \end{bmatrix}$$

Complete the following pseudocode for determining these classifications by filling the boxes with the appropriate values.

Hint: You may use a function called `sign(x)`*, which returns* -1 *for* $x < 0$*,* 0 *for* $x = 0$*, and* 1 *for* $x > 0$*.*

Take the transpose() of θ to get θ^T.

Take the dot product() between θ^T and x, and offset each value in the resulting matrix with θ_0 to get $\theta^T x + \theta_0$

Take the sign() of $\theta^T x + \theta_0$

◀ $\theta^T x + \theta_0$ | $\theta x + \theta_0$ | θ | θ^T | θ_0 | $x\theta^T + \theta_0$ ▶

Save | Submit | View Answer | Ask for Help 100.00%

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Meta 6.036 Lab 1

- Transformer
- Objectives
  - Masked prediction
  - Sequential

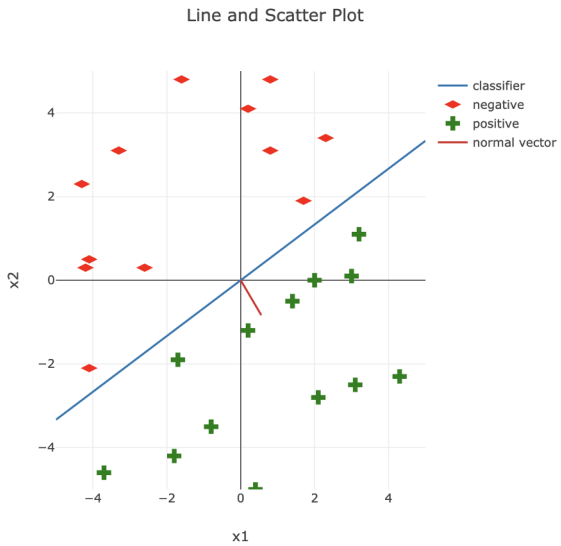
Massachusetts Institute of Technology

Meta 6.036 Lab 1

1.4) Visualizing and Building Intuition

For the questions below, please use the visualization tool below.

θ_1 2 θ_2 -3 θ_0 0



Probabilistic Programming Example

probabilistic program	probabilistic outcomes
var a = flip(0.3)	1 0 0 1 · ·
var b = flip(0.3)	0 0 0 0 · ·
var c = flip(0.3)	1 0 1 0 · ·
return a+b+c	2 0 1 1 · ·

probability

Probabilistic Programming Example

probabilistic program

```
infer(
 function() {
  var a = flip(0.3)
  var b = flip(0.3)
  var c = flip(0.3)
  condition(a+b==1)
  return a+b+c })
```

probabilistic outcomes

1	0	0	1 . .
0	0	0	0 . .
1	0	1	0 . .
T	F	F	T
2	0	1	1 . .

infer a+b+c | a+b=1

rejection sampling: run program and reject return values that do not satisfy constraints (inefficient)